Name ANSWERS	Class

MATHS TEACHER HUB

www.MathsTeacherHub.com

Cones, spheres and pyramids

(9 – 1) Topic booklet

Higher

These questions have been collated from previous years GCSE Mathematics papers.

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.	Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- If the question is a **1H** question you are not allowed to use a calculator.
- If the question is a **2H** or a **3H** question, you may use a calculator to help you answer.

Information

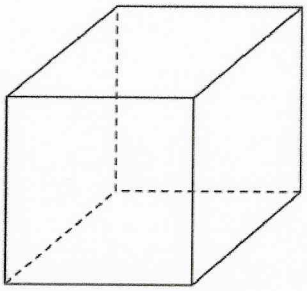
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

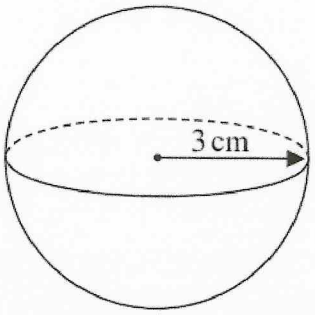
Advice

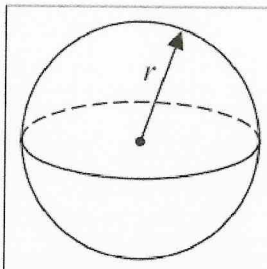
- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions
Write your answers in the space provided.
You must write down all the stages in your working.

7 The diagram shows a cube with edges of length x cm and a sphere of radius 3 cm.







Surface area of sphere $= 4\pi r^2$

The surface area of the cube is equal to the surface area of the sphere.

Show that $x = \sqrt{k\pi}$ where k is an integer.

Sphere $= 4 \times \pi \times 3^2$

$= 4 \times \pi \times 9$

$= 36\pi$

Cube $= 36\pi \div 6$

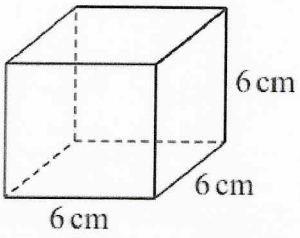
$= 6\pi$

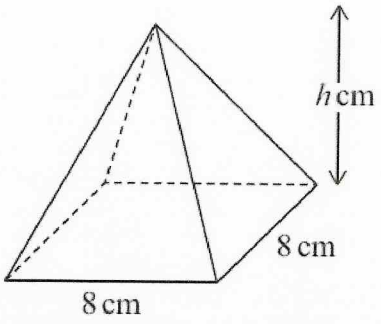
$\boxed{\frac{6\pi}{x}}$

$\sqrt{6\pi} = x$

(Total for Question 7 is 4 marks)

9 The diagram shows a cube and a square-based pyramid.





The volume of the cube is equal to the volume of the pyramid.

Work out the perpendicular height, h cm, of the pyramid.

Volume of cube $= 6^3$

$= 216 \text{cm}^3$

Volume of pyramid $= \frac{1}{3} \times (\text{base}) \times \text{height}$

$216 = \frac{1}{3} \times 64 \times h$

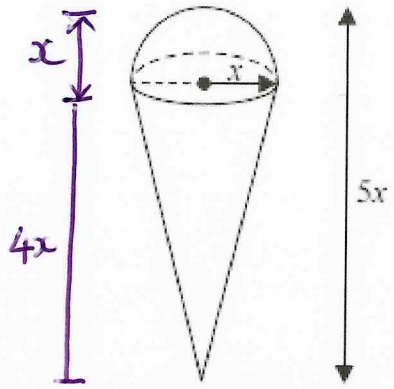
$648 = 64 \times h$

$10.125 = h$

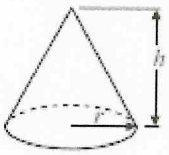
10.125 ..... cm

(Total for Question 9 is 3 marks)

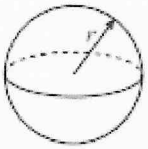
11 A solid is made by putting a hemisphere on top of a cone.



Volume of cone $= \frac{1}{3}\pi r^2 h$

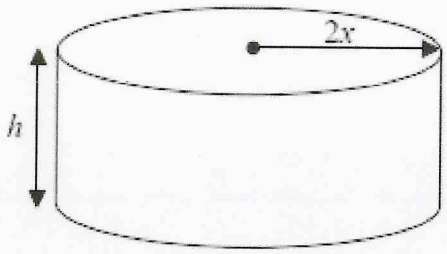


Volume of sphere $= \frac{4}{3}\pi r^3$



Volume of hemisphere $= \frac{4}{6}\pi r^3$

The total height of the solid is $5x$
The radius of the base of the cone is x
The radius of the hemisphere is x



A cylinder has the same volume as the solid.
The cylinder has radius $2x$ and height h
All measurements are in centimetres.

Find a formula for h in terms of x
Give your answer in its simplest form.

Cone

$\frac{1}{3} \times x^2 \times \pi \times 4x = \frac{4}{3}x^3\pi$

Hemisphere

$\frac{4}{6} \times \pi \times x^3 = \frac{2}{3}x^3\pi$

Solid shape

$\frac{4}{3}x^3\pi + \frac{2}{3}x^3\pi = 2x^3\pi$

Cylinder -

$\pi \times (2x)^2 \times h = 4x^2\pi h$

$\div \pi$ | $2x^3\pi = 4x^2\pi h$ | $\div \pi$

$\div x^2$ | $2x^3 = 4x^2 h$ | $\div x^2$

$\div 4$ | $2x = 4h$ | $\div 4$

$\frac{2x}{4} = h$

$\frac{1}{2}x = h$

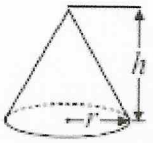
$h = \frac{1}{2}x$

(Total for Question 11 is 5 marks)

15 A cone has a volume of 98 cm^3. (100cm^3)
The radius of the cone is 5.13 cm. (5cm)

(a) Work out an estimate for the height of the cone.

Volume of cone = $\frac{1}{3}\pi r^2 h$



$V = \frac{1}{3} \times \pi \times r^2 \times h$

$100 = \frac{1}{3} \times 3 \times 5^2 \times h$

$\frac{100}{25} = h$

$h = 4$ cm

(3)

John uses a calculator to work out the height of the cone to 2 decimal places.

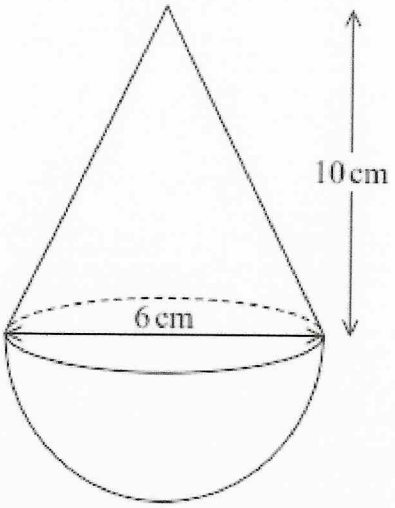
(b) Will your estimate be more than John's answer or less than John's answer?
Give reasons for your answer.

The height will be smaller, so the estimate will be more. π and the radius were rounded down.

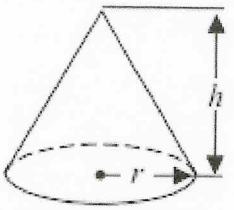
(1)

May 2017 – Paper 1H **(Total for Question 15 is 4 marks)**

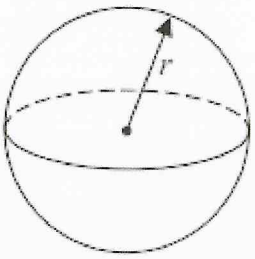
15 The diagram shows a solid shape.
The shape is a cone on top of a hemisphere.



Volume of a cone $= \frac{1}{3}\pi r^2 h$



Volume of a sphere $= \frac{4}{3}\pi r^3$



Volume of a hemisphere $= \frac{4}{6}\pi r^3$

The height of the cone is 10 cm.
The base of the cone has a diameter of 6 cm.
The hemisphere has a diameter of 6 cm.

The total volume of the shape is $k\pi$ cm^3, where k is an integer.

Work out the value of k.

Cone $\frac{1}{3} \times (\pi \times 3^2) \times 10 = 30\pi$

Hemisphere $\frac{4}{6} \times \pi \times 3^3 = \frac{2}{3} \times \pi \times 27 = 18\pi$

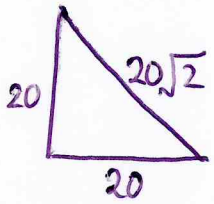
Total volume $= 30\pi + 18\pi$

$= 48\pi \text{ cm}^3$

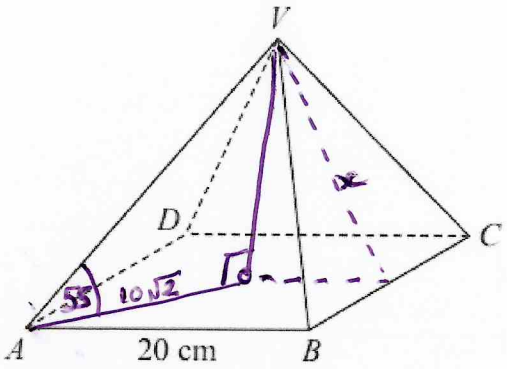
$k =$ 48

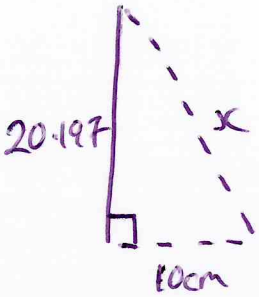
(Total for Question 15 is 4 marks)

16 *VABCD* is a solid pyramid.



$\sqrt{20^2+20^2}$ =





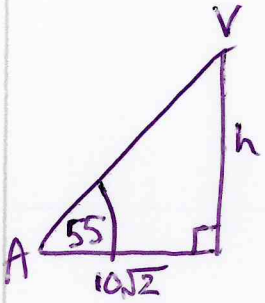
ABCD is a square of side 20 cm.

The angle between any sloping edge and the plane *ABCD* is 55°

Calculate the surface area of the pyramid.
Give your answer correct to 2 significant figures.

$x = \sqrt{20.197^2+10^2}$

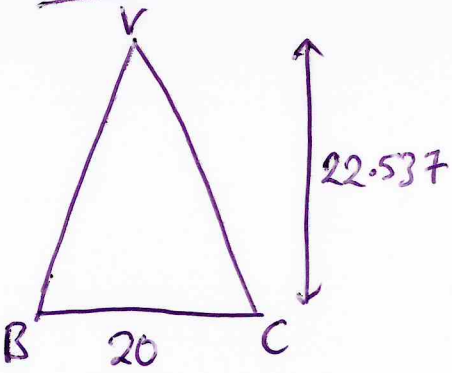
$x = 22.537\,11\text{cm}$

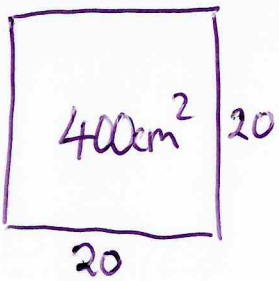


$h = \tan(55) \times 10\sqrt{2}$

$= 20.197\,0628\text{cm}$

Surface area





$225.37 \times 4 + 400 = 1301.4844$

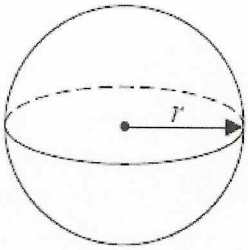
$\frac{22.537 \times 20}{2} = 225.37$

1300 cm²

Sample 1 – Paper 3H

(Total for Question 16 is 5 marks)

16 Here is a sphere.



Surface area of sphere = $4\pi r^2$

$\frac{3}{8}$ of the surface area of this sphere is $75\pi\,\text{cm}^2$

Find the diameter of the sphere.
Give your answer in the form $a\sqrt{b}$ where a is an integer and b is a prime number.

$\frac{1}{8}$ of surface area $= 25\pi$

$\frac{8}{8}$ of surface area $= 200\pi$

Surface Area $= 4 \times \pi \times r^2$

$\div\pi$ | $200\pi = 4 \times \pi \times r^2$ | $\div\pi$

$\div 4$ | $200 = 4r^2$ | $\div 4$

$\sqrt{\ }$ | $50 = r^2$ | $\sqrt{\ }$

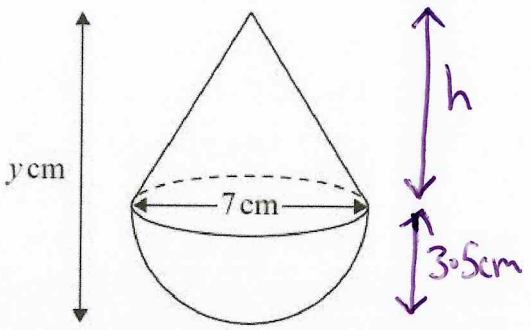
$\sqrt{50} = r$

$5\sqrt{2} = r$

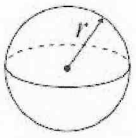
Diameter $10\sqrt{2}$ cm

(Total for Question 16 is 4 marks)

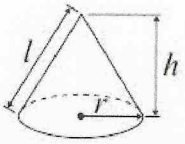
17 A solid cone is joined to a solid hemisphere to make the solid **T** shown below.



Volume of sphere $= \frac{4}{3}\pi r^3$



Volume of cone $= \frac{1}{3}\pi r^2 h$



The diameter of the base of the cone is 7 cm.
The diameter of the hemisphere is 7 cm.

Volume of hemisphere $= \frac{4}{6}\pi r^3$

The total volume of **T** is 120π cm^3
The total height of **T** is y cm.

(a) Calculate the value of y.
Give your answer correct to 3 significant figures.

$120\pi = \left(\frac{4}{6} \times \pi \times 3.5^3\right) + \left(\frac{1}{3} \times \pi \times 3.5^2 \times h\right)$

$120\pi - \left(\frac{4}{6} \times \pi \times 3.5^3\right) = \frac{1}{3} \times \pi \times 3.5^2 \times h$

$22.38775 = h$

$y = 22.38775 + 3.5 = 25.887755$

$y =$ 25.9

(4)

The diameter of the base of the cone and the diameter of the hemisphere are both increased by the same amount.
Assuming the total volume of **T** does not change,

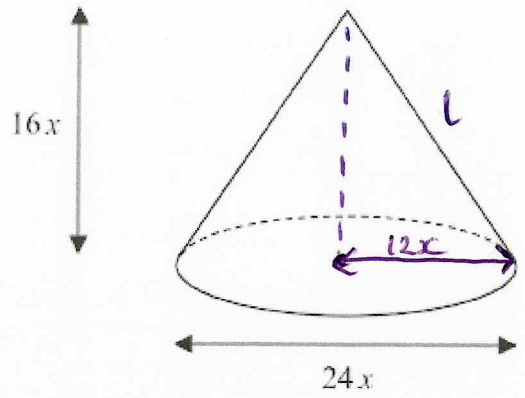
(b) explain the effect this would have on your answer to part (a).

The height of the cone would be shorter

(1)

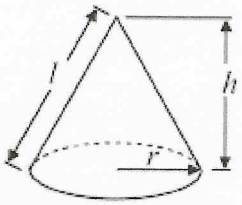
(Total for Question 17 is 5 marks)

17 The diagram shows a solid cone.



Volume of cone $= \frac{1}{3}\pi r^2 h$

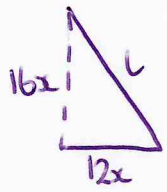
Curved surface area of cone $= \pi r l$



The diameter of the base of the cone is $24x$ cm.
The height of the cone is $16x$ cm.

The curved surface area of the cone is 2160π cm^2.
The volume of the cone is $V\pi$ cm^3, where V is an integer.

Find the value of V.



$l = \sqrt{12^2 + 16^2}$

$l = 20x$

Curved surface Area $= \pi \times r \times l$

$2160\pi = \pi \times 12x \times 20x$

$2160 = 240x^2$

$9 = x^2$

$3 = x$

Volume

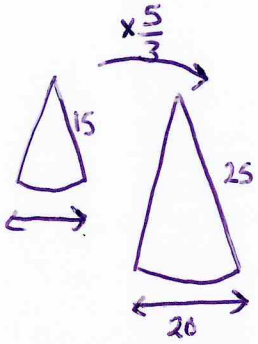
$\frac{1}{3} \times \pi \times 36^2 \times 48 = 20736\pi$

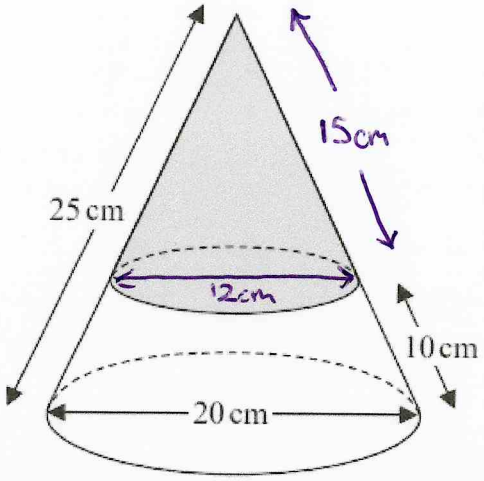
$= 65144.065$

20736

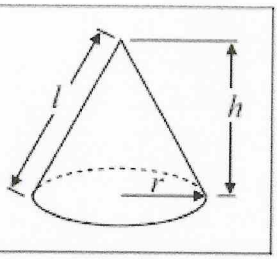
(Total for Question 17 is 5 marks)

18 The diagram represents a solid cone.





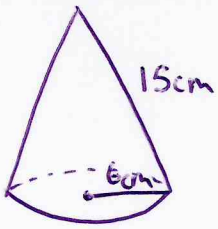
Curved surface area of cone = πrl



The cone has a base diameter of 20 cm and a slant height of 25 cm.

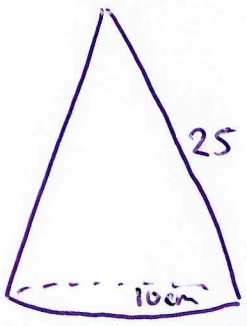
A circle is drawn around the surface of the cone at a slant height of 10 cm above the base.
The curved surface of the cone above the circle is painted grey.

Work out the area of the curved surface of the cone that is **not** painted grey.
Give your answer as a multiple of π
You must show all your working.



$\pi \times 6 \times 15$

$= 90\pi$



$\pi \times 10 \times 25$

$= 250\pi$

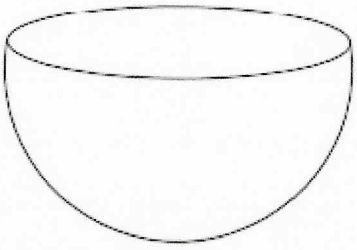
$250\pi - 90\pi$

$= 160\pi$

160π cm^2

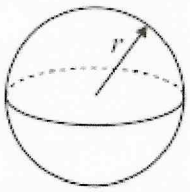
(Total for Question 18 is 4 marks)

18 The diagram shows a solid hemisphere.



Volume of sphere $= \frac{4}{3}\pi r^3$

Surface area of sphere $= 4\pi r^2$



The volume of the hemisphere is $\frac{250}{3}\pi$

Work out the exact total surface area of the solid hemisphere.
Give your answer as a multiple of π.

Volume of hemisphere $= \frac{4}{6}\pi r^3$

Surface area hemisphere $= 2\pi r^2$

$\frac{2}{3}\pi r^3 = \frac{250}{3}\pi$

$\div \pi$ $\quad$ $\div \pi$

$\frac{2}{3}r^3 = \frac{250}{3}$

$\times 3$ $\quad$ $\times 3$

$2r^3 = 250$

$\div 2$ $\quad$ $\div 2$

$r^3 = 125$

$\sqrt[3]{}$ $\quad$ $\sqrt[3]{}$

$r = 5$

Surface area

$2\pi \times 5^2 + \pi \times 5^2$

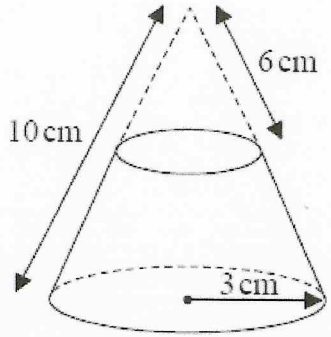
$= 50\pi + 25\pi$

$= 75\pi$

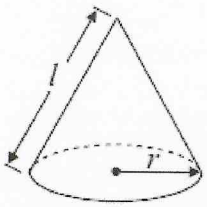
75 ..... cm^2

(Total for Question 18 is 4 marks)

18 A solid frustum is made by removing a small cone from a large cone as shown in the diagram.

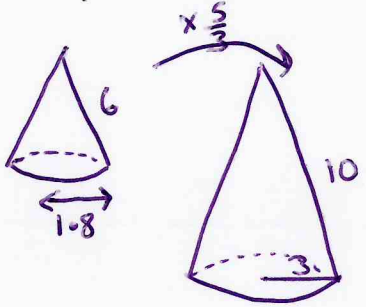


Curved surface area of cone = πrl



The slant height of the small cone is 6 cm.
The slant height of the large cone is 10 cm.
The radius of the base of the large cone is 3 cm.

Calculate the total surface area of the frustum.
Give your answer correct to 3 significant figures.



$(\pi \times 3 \times 10) - (\pi \times 1.8 \times 6)$

$= 30\pi - = 10.8\pi$

$= 60.318578955\text{cm}^2$

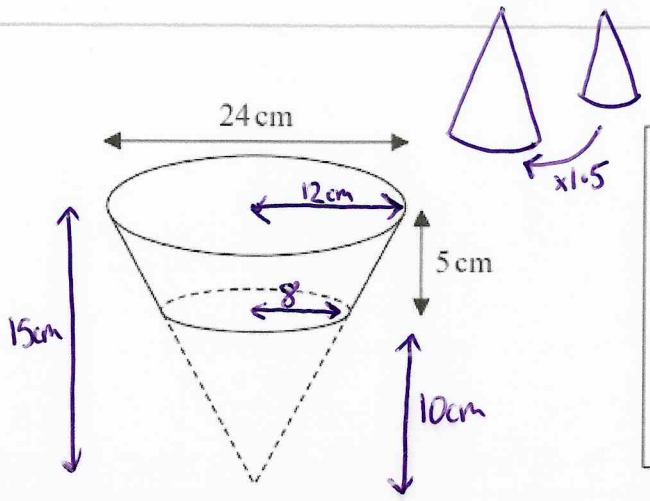
Curved area + top circle + bottom circle

$60.31857 + \pi \times 1.8^2 + \pi \times 3^2 = 98.77167303$

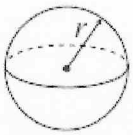
98.8 cm²

(Total for Question 18 is 5 marks)

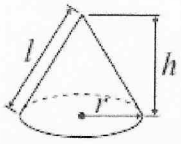
19



Volume of sphere $= \frac{4}{3}\pi r^3$



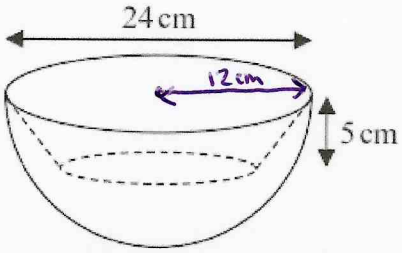
Volume of cone $= \frac{1}{3}\pi r^2 h$



Volume of Hemisphere $= \frac{4}{6}\pi r^3$

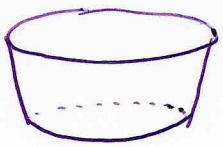
The diagram above shows a frustum **F** of a cone.
The frustum is made by removing a cone with height 10 cm from a solid cone with height 15 cm and base diameter 24 cm.

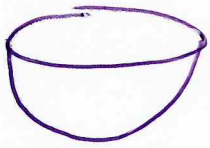
The solid **S** is made by removing **F** from a solid hemisphere as shown in the diagram below.



The hemisphere has diameter 24 cm.

Calculate the volume of solid **S**.
Give your answer correct to 3 significant figures.

Volume of frustum  $= \left(\frac{1}{3}\pi \times 12^2 \times 15\right) - \left(\frac{1}{3}\pi \times 8^2 \times 10\right) = \frac{1520}{3}\pi$

Volume of Hemisphere  $= \frac{2}{3} \times \pi \times 12^3 = 1152\pi$

Volume of S $= 1152\pi - \frac{1520}{3}\pi$

$= \frac{1936}{3}\pi$

$$\frac{1936}{3}\pi = 2027.3744\,59$$

2030 ..... cm^3

(Total for Question 19 is 4 marks)

19 **A**, **B** and **C** are three spheres.

The volume of sphere **A** is $125\,cm^3$
The volume of sphere **B** is $27\,cm^3$

B : C
1 : 2
3 : 6

The ratio of the radius of sphere **B** to the radius of sphere **C** is 1 : 2

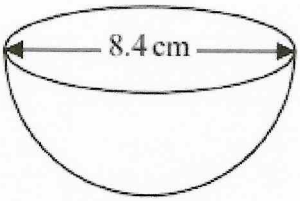
Work out the ratio of the surface area of sphere **A** to the surface area of sphere **C**.

	A	B	C
linear	5	3	6
Area	25	9	36
Volume	125	27	216

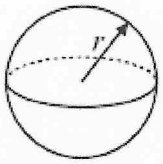
25 : 36

(Total for Question 19 is 3 marks)

19 The diagram shows a hemisphere with diameter 8.4 cm.



Volume of sphere $= \frac{4}{3}\pi r^3$



Work out the volume of the hemisphere.
Give your answer correct to 3 significant figures.

Volume of hemisphere $= \frac{4}{6}\pi r^3$

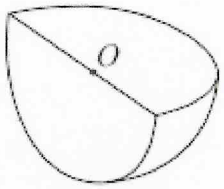
$V = \frac{4}{6} \times \pi \times 4.2^3$

$= 155.1695443 \text{ cm}^3$

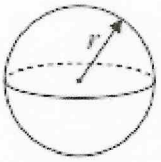
155 cm^3

(Total for Question 19 is 2 marks)

19 Shape **S** is one quarter of a solid sphere, centre O.



Shape **S**

Volume of sphere $= \frac{4}{3}\pi r^3$	
Surface area of sphere $= 4\pi r^2$	

The volume of **S** is 576π cm^3

Find the surface area of **S**.
Give your answer correct to 3 significant figures.
You must show your working.

Volume

S = $\frac{1}{4}$ of a sphere

$576\pi \times 4 = 2304\pi$

Volume $= \frac{4}{3} \times \pi \times r^3$

$2304\pi = \frac{4}{3} \times \pi \times r^3$

$2304 = \frac{4}{3} \times r^3$

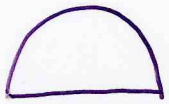
$6912 = 4 \times r^3$

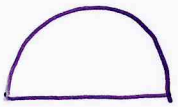
$1728 = r^3$

$12 = r$

Surface area

$\frac{4 \times \pi \times 12^2}{4} = 144\pi$

+  $\frac{\pi \times 12^2}{2}$

+  $\frac{\pi \times 12^2}{2}$

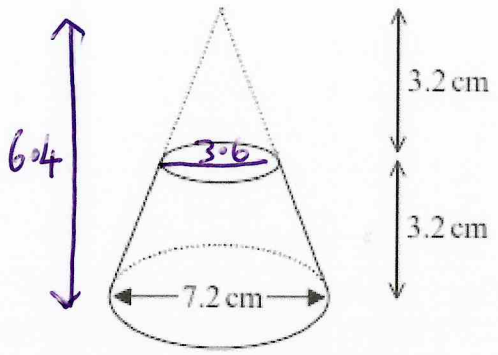
$= 288\pi$

$= 904.7786842$

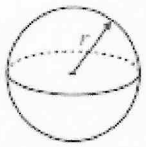
905 cm^2

(Total for Question 19 is 5 marks)

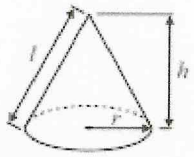
20 Here is a frustum of a cone.



Volume of sphere = $\frac{4}{3}\pi r^3$



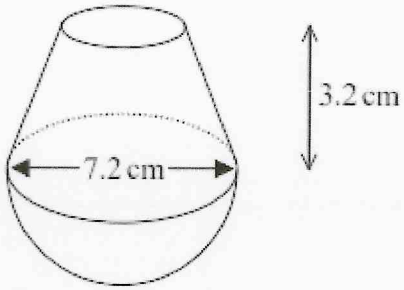
Volume of cone = $\frac{1}{3}\pi r^2 h$



The diagram shows that the frustum is made by removing a cone with height 3.2 cm from a solid cone with height 6.4 cm and base diameter 7.2 cm.

The frustum is joined to a solid hemisphere of diameter 7.2 cm to form the solid **S** shown below.

Solid **S**



The density of the frustum is 2.4 g/cm^3
The density of the hemisphere is 4.8 g/cm^3

Calculate the average density of solid **S**.

Volume of Frustum

$\left(\frac{1}{3}\times\pi\times 3.6^2\times 6.4\right)-\left(\frac{1}{3}\times\pi\times 1.8^2\times 3.2\right)$

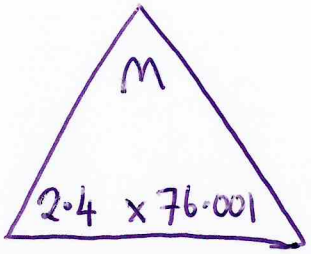
$= 76.00140948\text{cm}^3$

Volume of Hemisphere

$\frac{4}{6}\times\pi\times 3.6^3$

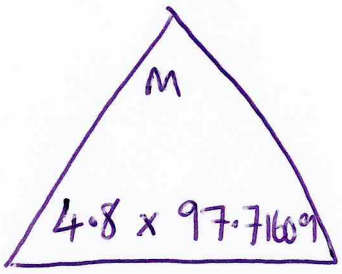
$= 97.7160979\text{cm}^3$

Frustum



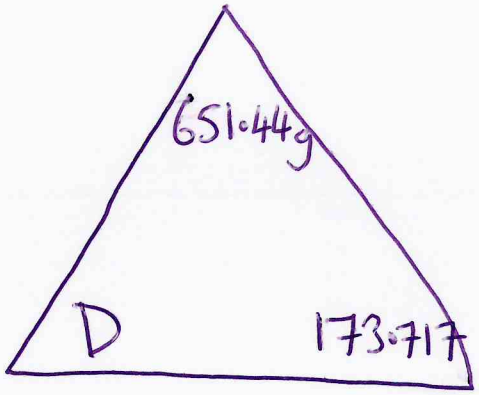
Mass = 182·4033g

Hemisphere



mass = 469·0372699g

Density of solid S

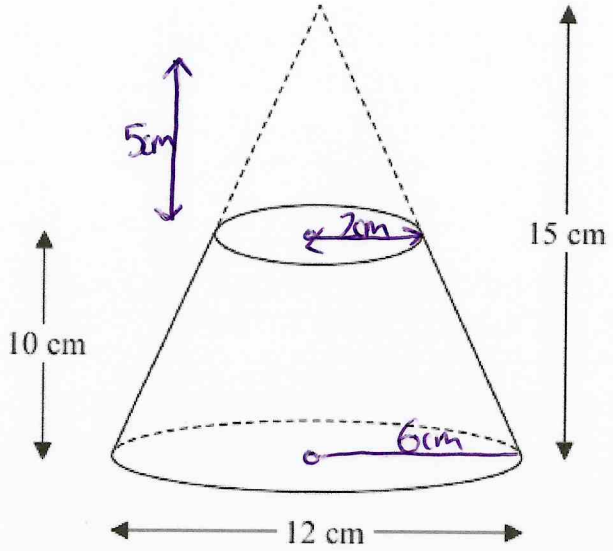


D = 3·750007196

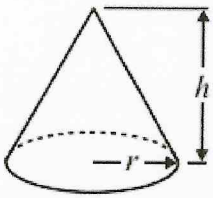
3·75 g/cm³

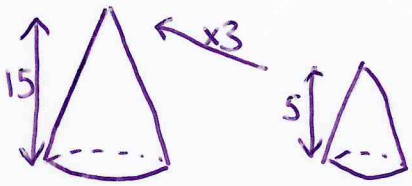
(Total for Question 20 is 5 marks)

22 A frustum is made by removing a small cone from a large cone as shown in the diagram.



Volume of cone $= \frac{1}{3}\pi r^2 h$

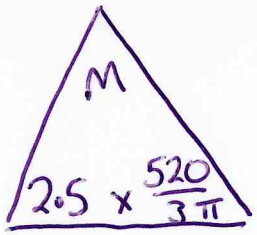




The frustum is made from glass.
The glass has a density of 2.5 g / cm^3

Work out the mass of the frustum.
Give your answer to an appropriate degree of accuracy.

Volume of Frustum $= \left(\frac{1}{3}\pi \times 6^2 \times 15\right) - \left(\frac{1}{3}\pi \times 2^2 \times 5\right) = \frac{520}{3}\pi$



Mass $= \frac{1300}{3}\pi = 1361.356817$g

1360
(3 significant figs)

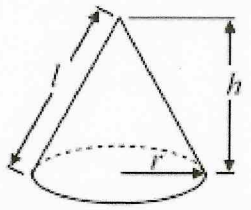
(Total for Question 22 is 5 marks)

23 The diagram shows a sector $OACB$ of a circle with centre O.
The point C is the midpoint of the arc AB.

The diagram also shows a hollow cone with vertex O.
The cone is formed by joining OA and OB.

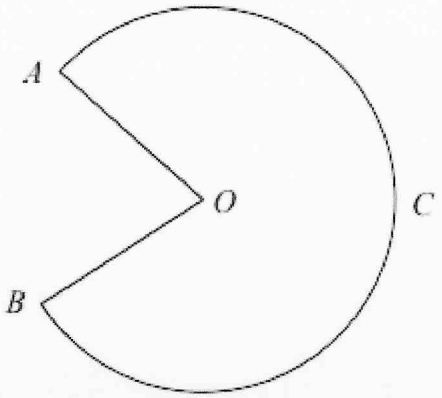
Volume of cone $= \frac{1}{3}\pi r^2 h$

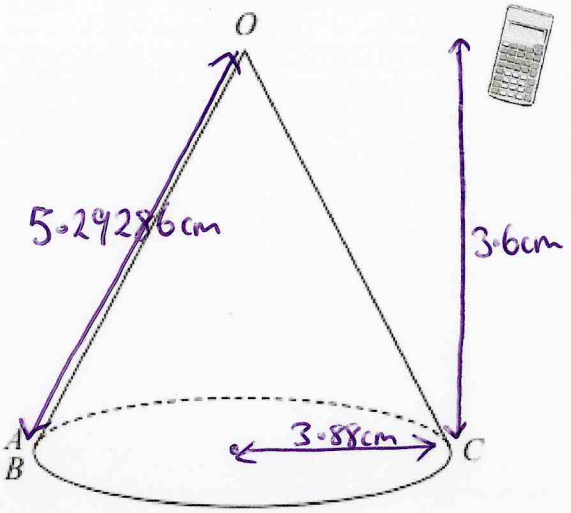
Curved surface area of cone $= \pi r l$



$OA = \sqrt{3.6^2 + 3.88^2}$

$= 5.29286\text{cm}$





The cone has volume $56.8\,\text{cm}^3$ and height $3.6\,\text{cm}$.

Calculate the size of angle AOB of sector $OACB$.
Give your answer correct to 3 significant figures.
You must show all your working.

Volume of Cone $= \frac{1}{3} \times \pi \times r^2 \times h$

$56.8 = \frac{1}{3} \times \pi \times r^2 \times 3.6$

$\sqrt{\frac{56.8}{1.2\pi}} = r$

$3.8815\text{cm} = r$

$$\text{ARC ACB} = \frac{\theta}{360} \times \pi \times d$$

$$24.38869 = \frac{\theta}{360} \times \pi \times 10.5857$$

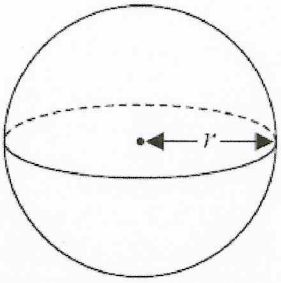
$$\frac{24.38869 \times 360}{10.5857\pi} = \theta$$

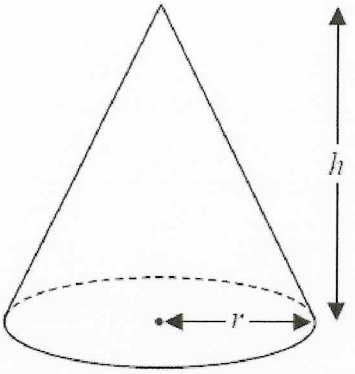
$$264.0100^\circ = \theta$$

264 °

(Total for Question 23 is 5 marks)

24 Here is a solid sphere and a solid cone.





Volume of sphere $= \frac{4}{3}\pi r^3$

Volume of cone $= \frac{1}{3}\pi r^2 h$

All measurements are in cm.

The volume of the sphere is equal to the volume of the cone.

(a) Find $r : h$
Give your answer in its simplest form.

$\frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$

$\times 3$ $\quad 4\pi r^3 = \pi r^2 h$ $\quad \times 3$

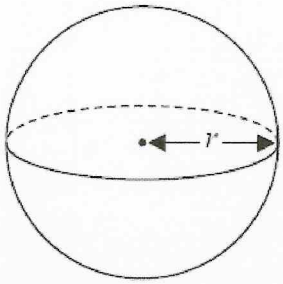
$\div \pi$ $\quad 4r^3 = r^2 h$ $\quad \div \pi$

$\div r^2$ $\quad 4r = h$ $\quad \div r^2$

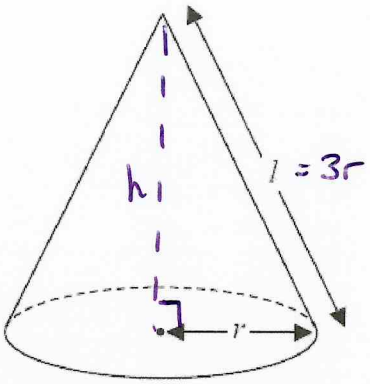
1 : 4

(2)

Here is a different solid sphere and a different solid cone.



Surface area of sphere = $4\pi r^2$



Curved area of cone = πrl

All measurements are in cm.

The surface area of the sphere is equal to the **total** surface area of the cone.

(b) Find $r : h$

Give your answer in the form $1 : \sqrt{n}$ where n is an integer.

$\div \pi$ | $4\pi r^2 = \pi rl + \pi r^2$ | $\div \pi$

$-r^2$ | $4r^2 = rl + r^2$ | $-r^2$

$\div r$ | $3r^2 = rl$ | $\div r$

$3r = l$

$3r = \sqrt{r^2 + h^2}$

$9r^2 = r^2 + h^2$

$8r^2 = h^2$

$\sqrt{8}r = h$

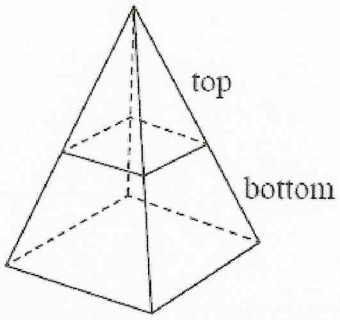
$1 : \sqrt{8}$

(4)

(Total for Question 24 is 6 marks)

$r = 1 \quad h = \sqrt{8}$

25 The pyramid **P** is formed from two parts made of different materials.



The top part of **P** has a mass of 92.8 g and is made from material with a density of 2.9 g/cm^3
The bottom part of **P** has a mass of 972.8 g
The average density of **P** is 4.7 g/cm^3

Calculate the volume of the top part of **P** as a percentage of the total volume of **P**.
Give your answer correct to 1 decimal place.
You must show all your working.

Volume of top $= \frac{\text{mass}}{\text{density}} = \frac{92.8}{2.9} = 32\,\text{cm}^3$

Total Volume $= \frac{\text{total mass}}{\text{average density}} = \frac{972.8 + 92.8}{4.7} = 226.723404\,\text{cm}^3$

$\frac{32}{226.723404} \times 100 = 14.114\%$

14.1 %

(Total for Question 25 is 5 marks)